Observational, Radiative and Dynamical Implications of Scale Invariance Near the Tropical Tropopause

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Scaling Exponent $H_1$

- Start with signal $f(t)$.
- The first order structure function associated with $f$ is

$$S_1(r; f) = \langle |f(t + r) - f(t)| \rangle$$

where the independent variable $r$ is called the “lag”.
- Plot $\log(S_1)$ vs. $\log(r)$. If the points describe a relatively straight line, then $f$ scales, and $H_1$ is given by the slope of the line.
- $H_1$ ranges from 0 to 1.
  - Values near 1: “persistence”, positive neighbor-to-neighbor correlation.
  - Values near 0: “anti-persistence”, negative neighbor-to-neighbor correlation.
H₁ for Air Along Aircraft Flight Tracks

- H₁ is related to spectral exponent β by β = 2H₁ + 1.
- Kolmogorov theory predicts β = 5/3 for the atmosphere in the horizontal direction, corresponding to Hₕ = 1/3.
- Bolgiano’s arguments predict β = 11/5 in the vertical direction, corresponding to Hᵥ = 3/5.
- Even in “level” flight, an aircraft samples a mix of the horizontal and vertical structure of the atmosphere.
- The scaling exponent along an aircraft flight track is given by Hₜ = Hₕ/Hᵥ = 5/9 ≈ 0.56.
- Empirically, we see a slightly lower value, possibly due to the inertia of the aircraft. (Under investigation.)
WAM WB57F 19980511

Tropospheric Segment

![Graph of ozone concentration over time in the tropospheric segment.](image)

Log10(Moment) vs. Log10(Time Interval Length)

- $H_1 = 0.39 \pm 0.05$

Stratospheric Segment

![Graph of ozone concentration over time in the stratospheric segment.](image)

Log10(Moment) vs. Log10(Time Interval Length)
Synthetic Signal

$\alpha = 1.66 \pm 0.12$

$H_1 = 0.50 \pm 0.03$

$C_1 = 0.025 \pm 0.002$
Synthetic Signal Plus 10% Gaussian Noise

\[ \log(K(q, \eta)) \]

\[ \alpha = 1.64 \pm 0.17 \]

\[ C_1 = 0.027 \pm 0.002 \]

\[ H_1 = 0.29 \pm 0.02 \]

\[ H_1 = 0.02 \pm 0.02 \]
WAM WB57F 19980501 MTP Isentrope Curtain

The graph shows the progression of pressure altitude over universal time for different isentropes. The markers indicate the tropopause and aircraft altitude.
WAM WB57F 19980501 minlat-begindescent

- Water (ppmv)
  - Time (UTC Seconds)
  - Log10(Moment)
  - Log10(Time Interval Length)
  - PDF
  - Water (ppmv) Histogram

- Ozone (ppbv)
  - Time (UTC Seconds)
  - Log10(Moment)
  - Log10(Time Interval Length)
  - PDF
  - Ozone (ppbv) Histogram

H1 = 0.30±0.07
H1 = 0.35±0.07
$WAM\ WB57F\ 19980501\ minlat$-begin
descent

$H_1 = 0.65 \pm 0.06$

$H_1 = 0.70 \pm 0.02$

PDF

PDF
19990921 Scaling Exponents

Wind Speed
\( H_1 = 0.48 \pm 0.04 \)

Temperature
\( H_1 = 0.55 \pm 0.04 \)

Ozone
\( H_1 = 0.34 \pm 0.04 \)

Total H\(_2\)O
\( H_1 = 0.31 \pm 0.05 \)
## Flight Segments Included In Analysis

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>Date yyyyymmdd</th>
<th>Segment duration (s)</th>
<th>Coordinates (ºlat, ºlon)</th>
<th>Segment center time (UTCs)</th>
<th>Approximate mean θ (K)</th>
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<tbody>
<tr>
<td>ER-2</td>
<td>19870203</td>
<td>6971</td>
<td>(14ºS,129ºE) (18ºS, 117ºE)</td>
<td>91635</td>
<td>412</td>
</tr>
<tr>
<td>ER-2</td>
<td>19870814</td>
<td>6845</td>
<td>(7ºN, 79ºW) (5ºS, 78ºW)</td>
<td>48470</td>
<td>391</td>
</tr>
<tr>
<td>ER-2</td>
<td>19941026</td>
<td>8207</td>
<td>(15ºN, 158ºW)(1ºN,159ºW)</td>
<td>76878</td>
<td>410</td>
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<td>WB57F</td>
<td>19980409</td>
<td>7017</td>
<td>(28ºN, 93ºW) (17ºN, 84ºW)</td>
<td>59534</td>
<td>368</td>
</tr>
<tr>
<td>WB57F</td>
<td>19980409</td>
<td>5943</td>
<td>(19ºN, 85ºW) (27ºN, 91ºW)</td>
<td>66911</td>
<td>413</td>
</tr>
<tr>
<td>WB57F</td>
<td>19980501</td>
<td>8209</td>
<td>(28ºN, 94ºW) (14ºN, 95ºW)</td>
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<tr>
<td>WB57F</td>
<td>19980501</td>
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<td>(13ºN, 95ºW) (26ºN, 95ºW)</td>
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<td>370</td>
</tr>
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</tr>
<tr>
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<td>19980511</td>
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<td>(28ºN, 95ºW) (11ºN, 95ºW)</td>
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<td>374</td>
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<td>20020726</td>
<td>3125</td>
<td>(22ºN, 86ºW) (17ºN, 84ºW)</td>
<td>63738</td>
<td>369</td>
</tr>
</tbody>
</table>
Calculations of $H_1$

<table>
<thead>
<tr>
<th>Segment duration(s)</th>
<th>$H_1$(H$_2$O)</th>
<th>$H_1$(O$_3$)</th>
<th>$H_1([u^2+v^2]^{1/2})$</th>
<th>$H_1$(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6971</td>
<td>-</td>
<td>0.34 ± 0.07</td>
<td>-</td>
<td>0.49 ± 0.03</td>
</tr>
<tr>
<td>6845</td>
<td>-</td>
<td>-</td>
<td>0.35 ± 0.04</td>
<td>0.34 ± 0.04</td>
</tr>
<tr>
<td>8207</td>
<td>-</td>
<td>0.37 ± 0.04</td>
<td>0.49 ± 0.07</td>
<td>0.52 ± 0.02</td>
</tr>
<tr>
<td>7017</td>
<td>-</td>
<td>-</td>
<td>0.56 ± 0.07</td>
<td>0.66 ± 0.02</td>
</tr>
<tr>
<td>5943</td>
<td>-</td>
<td>0.43 ± 0.05</td>
<td>0.36 ± 0.04</td>
<td>0.61 ± 0.03</td>
</tr>
<tr>
<td>8209</td>
<td>0.43 ± 0.05</td>
<td>0.51 ± 0.08</td>
<td>0.64 ± 0.06</td>
<td>0.65 ± 0.04</td>
</tr>
<tr>
<td>7242</td>
<td>0.30 ± 0.07</td>
<td>0.35 ± 0.07</td>
<td>0.65 ± 0.06</td>
<td>0.70 ± 0.02</td>
</tr>
<tr>
<td>7926</td>
<td>-</td>
<td>0.48 ± 0.10</td>
<td>0.36 ± 0.05</td>
<td>0.59 ± 0.03</td>
</tr>
<tr>
<td>5144</td>
<td>0.35 ± 0.05</td>
<td>0.51 ± 0.09</td>
<td>0.56 ± 0.03</td>
<td>-</td>
</tr>
<tr>
<td>8872</td>
<td>-</td>
<td>-</td>
<td>0.57 ± 0.06</td>
<td>0.65 ± 0.04</td>
</tr>
<tr>
<td>8052</td>
<td>-</td>
<td>0.39 ± 0.05</td>
<td>0.43 ± 0.04</td>
<td>0.62 ± 0.03</td>
</tr>
<tr>
<td>13525</td>
<td>0.26 ± 0.04</td>
<td>0.37 ± 0.06</td>
<td>-</td>
<td>0.47 ± 0.09</td>
</tr>
<tr>
<td>10992</td>
<td>0.31 ± 0.04</td>
<td>0.33 ± 0.04</td>
<td>0.48 ± 0.04</td>
<td>0.55 ± 0.04</td>
</tr>
<tr>
<td>3125</td>
<td>0.40 ± 0.04</td>
<td>0.43 ± 0.05</td>
<td>0.44 ± 0.08</td>
<td>0.47 ± 0.07</td>
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<tr>
<td>Mean</td>
<td>0.34 ± 0.06</td>
<td>0.41 ± 0.07</td>
<td>0.49 ± 0.11</td>
<td>0.56 ± 0.10</td>
</tr>
</tbody>
</table>
CRYSTAL-FACE 20020726 WB57

Horizontal Wind Speed (m/s)

Potential Temperature (K)

Time (seconds UTC)

370 K ± 5 K
20020726 Scaling Exponents

Wind Speed
$H_1 = 0.44 \pm 0.08$

Temperature
$H_1 = 0.47 \pm 0.07$

Ozone
$H_1 = 0.43 \pm 0.05$

H$_2$O Vapor
$H_1 = 0.40 \pm 0.04$
Summary

• Wind speed and temperature scale with $H_1 = 5/9$.
• Scale breaks for ozone and water on some flights at about 20 km appear to be instrumental.
• Total water, water vapor and ozone scale in the upper troposphere, as does carbon dioxide in the lower stratosphere, but not as passive scalars, i.e. $H_1 \neq 5/9$. Sources and sinks are operating faster than mixing.
Consequence

- Because the radiatively absorbing and emitting species scale, so must the energy input and output to the atmosphere: all scales play a role in forcing and dissipation.

Conclusions

- Better signal-to-noise ratio is required at high frequencies.
- Many more long, “constant level” flight tracks are needed in the troposphere, preferably with 10 Hz data.
Tropopause Potential Temperature vs. Latitude
AAOE ER-2 19870814 endascent-begininchgalt1

Wind Speed (m/s)

Temperature (K)

log$_{10}$(Moment)

log$_{10}$(Time Interval Length)

$H_1 = 0.35 \pm 0.04$

$H_1 = 0.34 \pm 0.04$

PDF

Wind Speed (m/s)

Temperature (K)
WAM WB57F 19980501 minlat-begindescent Ozone

\[ \alpha = 1.95 \pm 0.20 \]

\[ C_1 = 0.047 \pm 0.002 \]

\[ H_1 = 0.35 \pm 0.07 \]
WAM WB57F 19980501 minlat-begindescent Wind Speed

- Horizontal Wind Speed (m/s)
- Log($\eta$)
- $K(q)$
- Log($K(q_n)$)

$C_1 = 0.179 \pm 0.010$
$\alpha = 1.35 \pm 0.63$
$H_1 = 0.65 \pm 0.06$
WAM WB57F 19980501 minlat-begindescent Temperature

- Temperature (K)
  - Time (UTC Seconds)

- Log(\eta)
  - H1 = 0.70 ± 0.02

- K(q)
  - C1 = 0.061 ± 0.002

- Log(K(q, \eta))
  - \alpha = 1.72 ± 0.25
Scaling Exponents as a Function of Potential Temperature

- Temperature
- Wind speed
- Water
- Ozone
Water vapor and ozone along the Canberra flight track 55°N-68°N
AASE ER-2 19890207 takeoff-endascent
19890207 Back Trajectories at 390 K Potential Temperature
19620131 Potential Vorticity, 12 UTC, 375 K
19620201 Potential Vorticity, 12 UTC, 375 K
19620206 Potential Vorticity, 12 UTC, 375 K
WAM WB57F 19980507 endascent-begininchgalt, 370 K
ASHOE ER-2 19940327 Hawaii-Fiji

Methane (ppmv)

Water (ppmv)

Outer Tropics

Inner Tropics
Scaling Analysis of Airborne Observations Near the Tropopause

Adrian Tuck, Susan Hovde
NOAA Aeronomy Laboratory
Shaun Lovejoy
McGill University
Daniel Schertzer
Université Pierre et Marie Curie
$H_1$ Example (Synthetic Data)

$H_1 = \text{Slope} = 0.50 \pm 0.03$
Effect of Adding 10% Gaussian Noise

\[ \log( \langle |f(x+r) - f(x)|^2 \rangle ) \]

\[ H_1 = 0.29 \pm 0.02 \]

\[ H_1 = 0.02 \pm 0.02 \]
2.0
1.6
1.2
0.8
\log( |f(x+r) - f(x)| )

H_1 = 0.38 \pm 0.05

O_3 (ppbv)

Stratospheric Segment

\begin{align*}
\text{H}_1 &= 0.38 \pm 0.05 \\
\log(\langle |f(x+r) - f(x)| \rangle) &
\end{align*}

\begin{align*}
\log(\langle |f(x+r) - f(x)| \rangle) &
\end{align*}

\begin{align*}
\log(r) &
\end{align*}
Tropospheric Segment

\[ \log(|f(x+r) - f(x)|) \]

\[ H_1 = 0.69 \pm 0.12 \]

\[ H_1 = 0.15 \pm 0.03 \]